Upper-bounding Program Execution Time with Extreme Value Theory

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Sliding Focus /1

EVT → Use in Engineering sciences → Best/Worst Average Performance

Worst-Case Behaviour → MBPTA: Time Randomised Systems → Requirements

Deterministic Systems → Requirements → Tradeoffs
Extreme Value Theory (EVT)

- A methodology for predicting the occurrence of rare events
- For a given distribution of events (a population) the Central Limit Theory studies its bulk
- EVT studies its tail
  - Extreme deviations from the median of the probability distribution
- By analysing a sample of observations of the events of interest EVT determines the probability of extreme deviations to occur
- Widely used outside of computer science
Extreme Value Theory (EVT) /2

- EVT models the events of interest as random variables.
- Those events therefore have to be **independent** and **identically distributed** (i.i.d.).
  - Two random variables are said to be independent if they describe two events such that the occurrence of one does not have any impact on the occurrence of the other.
  - Two random variables are said to be identically distributed if they have the same probability distribution function.
- The system that produces those events must behave accordingly.
- Because of its founding hypotheses, EVT has no concern with the “representativeness” of the data passed to it.
When applied to the WCET problem, EVT computes a cumulative distribution function (or $p_{WCET}$) function that upper bounds the execution time of the program.

- Guaranteeing that it exceeds a given bound only with a probability lower than a given threshold.

- EVT is applied for measurement based timing analysis (aka MBTA).

- Independence holds here when it is not execution history that causes timing behaviour to jitter.

- Identical distribution holds here when the observations describe the same system under the same operating conditions.

  - For all inputs with bearing on the program’s timing behaviour:
    - Input vectors, initial state of hardware and software.
EVT in the WCET Context /2

- EVT is given in input a number of observations taken from real execution of the system of interest
  - Measurements runs of the program of interest taken under controlled analysis conditions

- EVT has nothing to say on the representativeness of those data, hence, on the safeness of the pWCET estimate that is computed from them
  - Low confidence: pWCET bounds only valid for the operating conditions used for the analysis
  - High confidence: the control exercised on the operating conditions allow firmer statements to be made on the safeness of the computed pWCET bounds
EVT in the WCET Context /3

• When using MBPTA, what can be said about the representativeness of the observations?
• Representativeness is determined by the quality of the data passed to EVT
• Or by the properties of the environment that produced those data
• Hence the pWCET estimates obtained with EVT-MBPTA are solely valid for the *sampled population*
• Or by extension, for the operating conditions subsumed by those data
• We need to understand what requirements emanate from these premises
The Goal of Applying EVT

- To compute $p_{\text{WCET}}$ estimates that hold under operating conditions that may occur during the actual execution of the system
  - Those conditions need not be exactly identical to those captured by the observation runs made at analysis time,
  - It suffices they represent them probabilistically

- Three ways to apply EVT to the WCET problem
  - A risky way, which exceeds in pragmatism (or lacks rigor)
  - An ideal way, which is unfeasible in practice (and which motivates the former pragmatism)
  - A more realistic middle-ground way, which requires understanding the operating conditions under which observations are made
Sliding Focus /2

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EVT: Observations and the System

System Under Analysis (SUA) → Observations \( \{o_i\} \)

Hardware and Software → Observations \( \{o_i\} \)

inputs
The Risky Way of Using EVT for WCET

- Says (s)he: ‘Let us collect observations by running the SUA for a while’
- We then get data for EVT by sampling the obtained observations
- And then apply the EVT method
- In that case the EVT results are only representative of the sampled population
- This fails to achieve the goal

Applying random sampling to a target population results in samples can be modeled with i.i.d variables

Regardless of the statistical representativeness of that to the real population!
The Ideal Way of Using EVT for WCET

• One might disregard the SUA and concentrate solely on the observations only if they had the *entire universe* of them at their disposal
  - You would pick at random from that *entire universe*
  - And then apply EVT to the resulting samples

• But you do not know this *population*
• Hence you cannot randomly sample from it
• This approach is obviously impossible in the general case
Sliding Focus /3

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- Requirements
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- Tradeoffs
A Realistic Way of Using EVT for WCET

• A possible solution is to look at the problem from the other end (the SUA)
  ➢ This requires first understanding and then – if possible – controlling the sources of variability in the outcome of observations

• This is the premise of Measurement-Based Probabilistic Timing Analysis (MBPTA)
In fact we are not interested in all elements of the universe of observations!

We can «help» the analysis procedure using block maxima and concentrate on the sub-universe of maximal elements.

Since we cannot tell that sub-universe apart a priori we have to gear the SUA so that it does produce them.

In doing so we must ensure that the SUA is set to operational conditions.
EVT in the WCET Context

Those should all be controlled
Those can be controlled
This is very hard to control
Time randomisation helps

- Lots of sources of variability are hard for the user to effectively control from outside of the SUA
- A number of them can be written off by enforcing constant-time behaviour at the lower tiers of the execution stack underneath the application
  - This can be done for the Operating System
  - This can be done by setting low-jitter processor resources to operate in worst-case mode
- Or else injecting time randomization in high-jitter execution resources
  - A change in the generation of latency not in the functional logic
EVT-MBPTA in a Nutshell

- **Procedure**
  - Observations
  - Sampling
  - Fitting
  - Comparison
  - Tail extension

- **Convergence**

\[
CRPS = \sum_{i=0}^{+\infty} [f_X(i) - f_Y(i)]^2
\]

- EVT fitting
- I.i.d tests
- Passed/not passed
- More runs

*ECRTS 2012*
Take-Home Message

• Extreme Value Theory is interesting
• It lends itself well to the WCET problem
• But its application needs (extreme 😊) care and attention

PROARTIS has pioneered an EVT-MBPTA method that works well with «PTA-friendly» processors and time-composable Operating Systems

• You may want to read about it at http://www.proartis-project.eu/publications
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