Upper-bounding Program Execution Time with Extreme Value Theory

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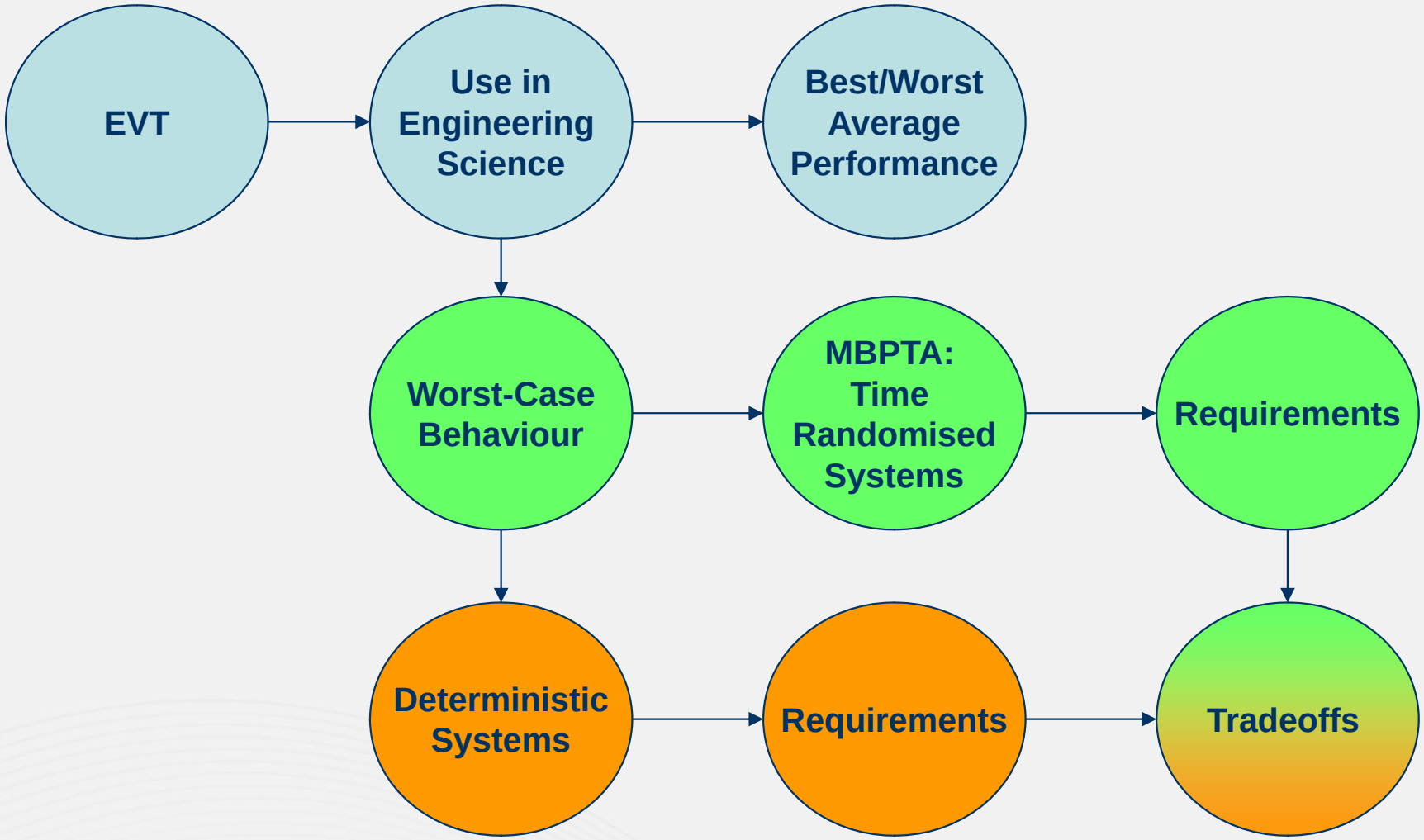
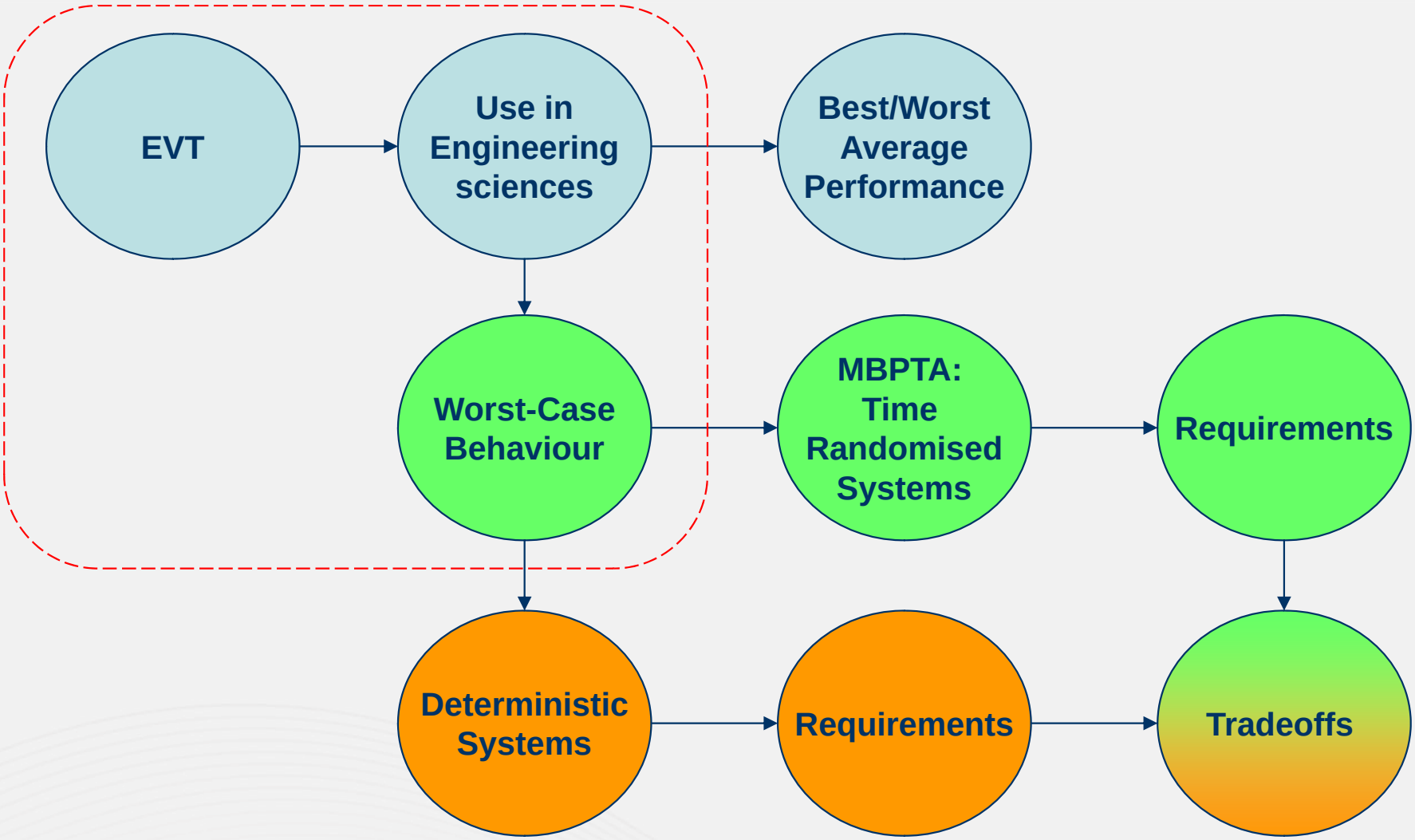
Outline


EVT
Use in Engineering Science
Best/Worst Average Performance
Worst-Case Behaviour
MBPTA: Time Randomised Systems
Requirements
Deterministic Systems
Requirements
Tradeoffs

Sliding Focus /1



Extreme Value Theory (EVT) /1

- *A methodology for predicting the occurrence of rare events*
- *For a given distribution of events (a population) the Central Limit Theory studies its bulk*
- *EVT studies its tail*
  - Extreme deviations from the median of the probability distribution
- *By analysing a **sample** of observations of the events of interest EVT determines the probability of extreme deviations to occur*
- *Widely used outside of computer science*

Extreme Value Theory (EVT) /2

- *EVT models the events of interest as random variables*
- *Those events therefore have to be **independent** and **identically distributed** (i.i.d.)*
  - Two random variables are said to be independent if they describe two events such that the occurrence of one does not have any impact on the occurrence of the other
  - Two random variables are said to be identically distributed if they have the same probability distribution function
- *The system that produces those events must behave accordingly*
- *Because of its founding hypotheses EVT has no concern with the "representativeness" of the data passed to it*

EVT in the WCET Context /1

- *When applied to the WCET problem, EVT computes a cumulative distribution function (or **pWCET**) function that upper bounds the execution time of the program*
  - Guaranteeing that it exceeds a given bound only with a probability lower than a given threshold
- *EVT is applied for measurement based timing analysis (aka **MBTA**)*
- ***Independence** holds here when it is not execution history that causes timing behaviour to jitter*
- ***Identical distribution** holds here when the observations describe the **same** system under the **same** operating conditions*
  - For all inputs with bearing on the program's timing behaviour
    - Input vectors, initial state of hardware and software

EVT in the WCET Context /2

- EVT is given in input a number of observations taken from real execution of the system of interest
  - *Measurements runs of the program of interest taken under **controlled** analysis conditions*

- *EVT has nothing to say on the **representativeness** of those data, hence, on the safeness of the **pWCET** estimate that is computed from them*
  - **Low confidence**: pWCET bounds only valid for the operating conditions used for the analysis
  - **High confidence**: the control exercised on the operating conditions allow firmer statements to be made on the safeness of the computed pWCET bounds

EVT in the WCET Context /3

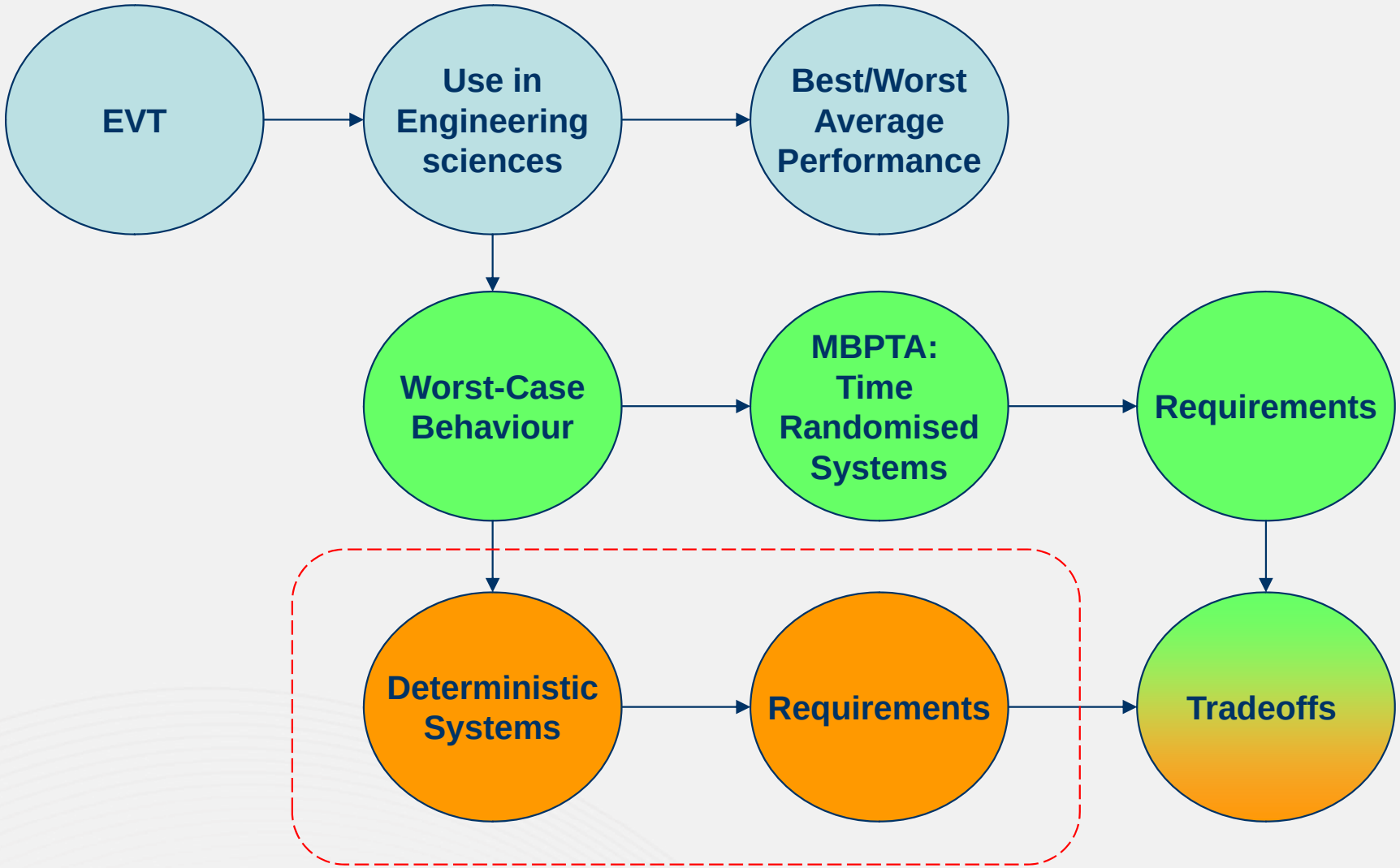
- *When using MBPTA, what can be said about the representativeness of the observations?*
- *Representativeness is determined by the quality of the data passed to EVT*
- ***Or** by the properties of the environment that produced those data*
- *Hence the pWCET estimates obtained with EVT-MBPTA are solely valid for the **sampled population***
- ***Or** by extension, for the operating conditions subsumed by those data*
- *We need to understand what requirements emanate from these premises*

The Goal of Applying EVT

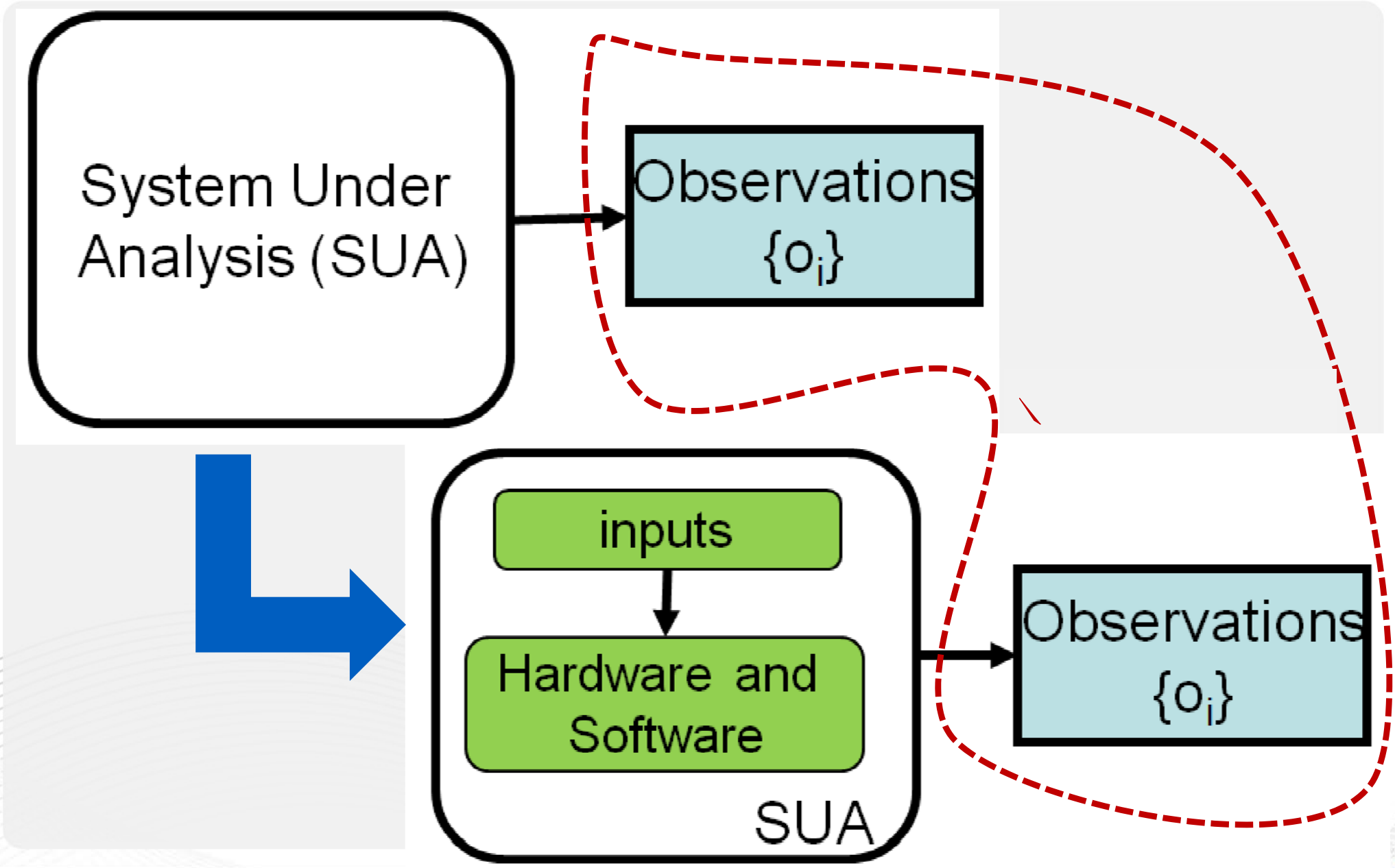
- *To compute pWCET estimates that hold under operating conditions that may occur during the actual execution of the system*
  - Those conditions need not be *exactly* identical to those captured by the observation runs made at analysis time,
  - It suffices they represent them probabilistically

- *Three ways to apply EVT to the WCET problem*
  - A **risky** way, which exceeds in pragmatism (or lacks rigor)
  - An **ideal** way, which is unfeasible in practice (and which motivates the former pragmatism)
  - A more **realistic** middle-ground way, which requires understanding the operating conditions under which observations are made

Sliding Focus /2

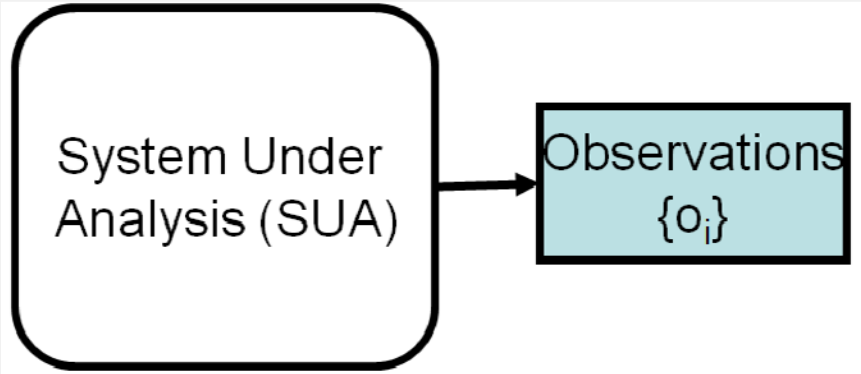


EVT: Observations and the System



The Risky Way of Using EVT for WCET

- *Says (s)he: 'Let us collect observations by running the SUA for a while'*
- *We then get data for EVT by sampling the obtained observations*
- *And then apply the EVT method*
- *In that case the EVT results are only representative of the sampled population*
- *This fails to achieve the goal*

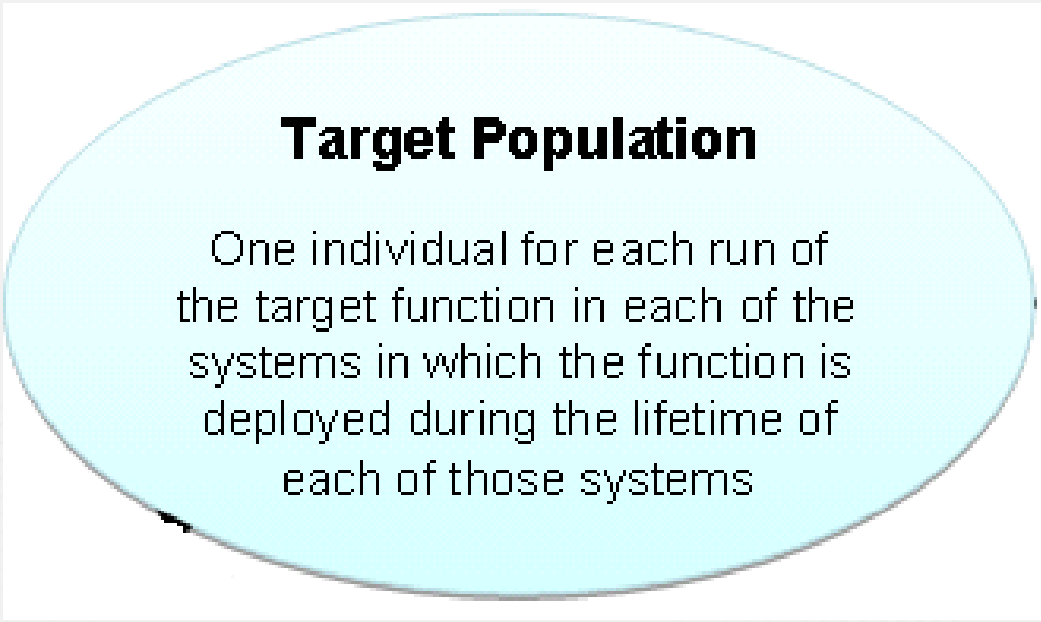


Applying random sampling to a target population results in samples can be modeled with i.i.d variables

Regardless of the statistical representativeness of that to the **real** population!

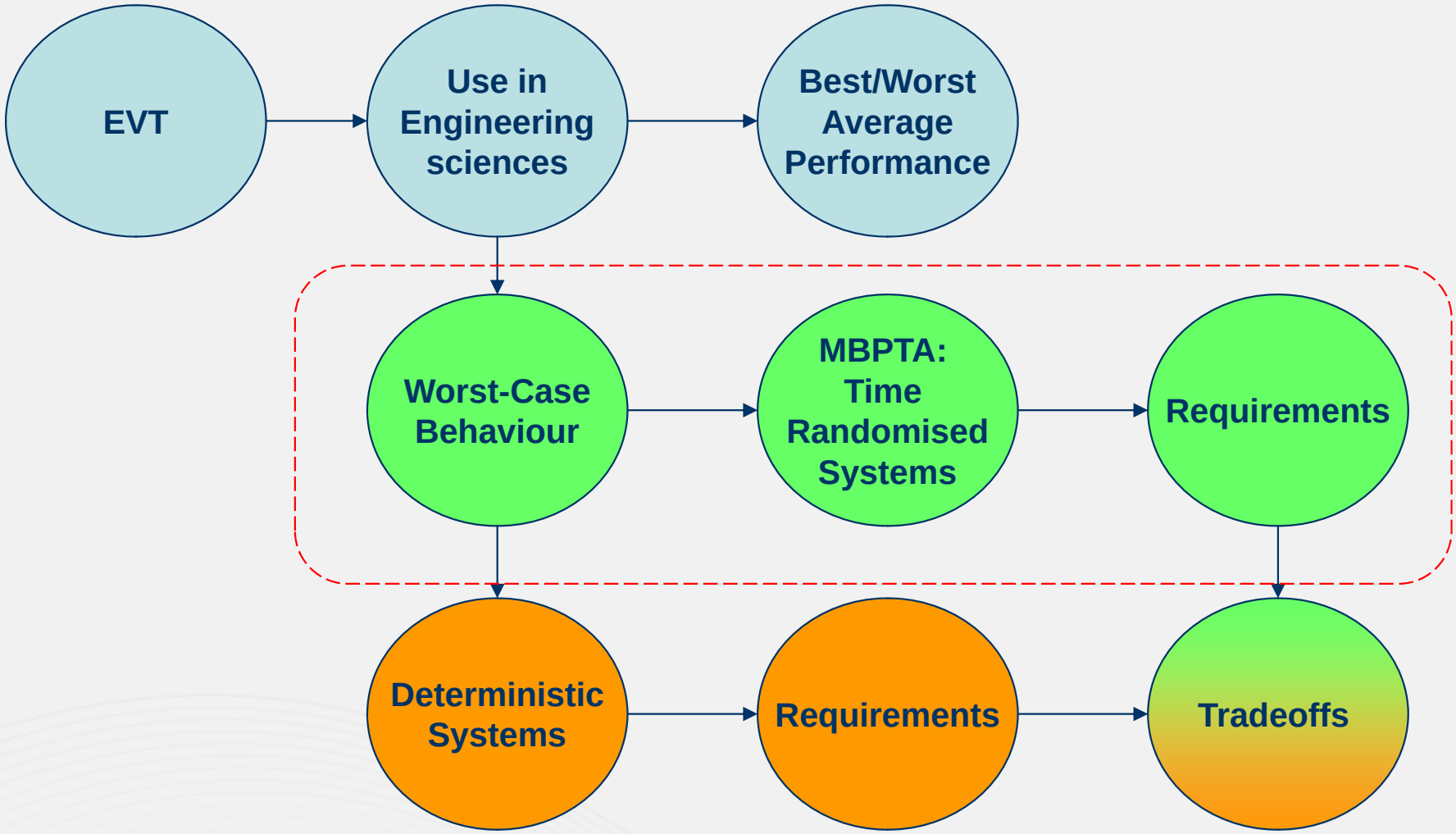
The Ideal Way of Using EVT for WCET

- *One might disregard the SUA and concentrate solely on the observations only if they had the **entire universe** of them at their disposal*
  - You would pick at random from that entire universe
  - And then apply EVT to the resulting samples



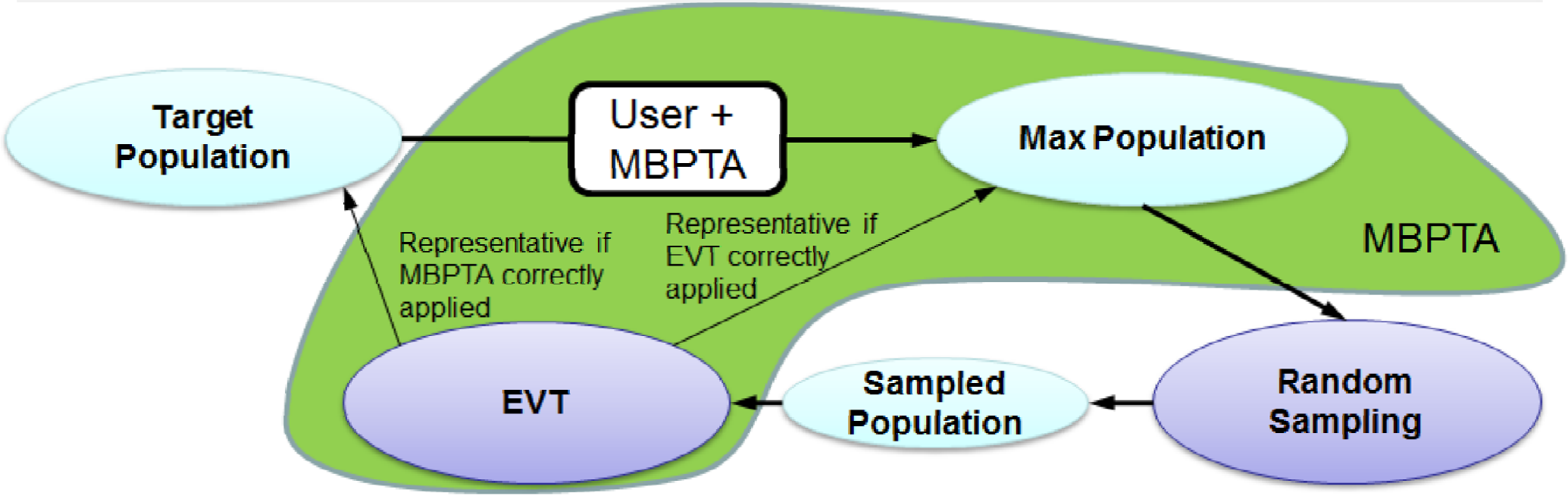
- *But you do not know this population*
- *Hence you cannot randomly sample from it*
- *This approach is obviously impossible in the general case*

Sliding Focus /3



A Realistic Way of Using EVT for WCET

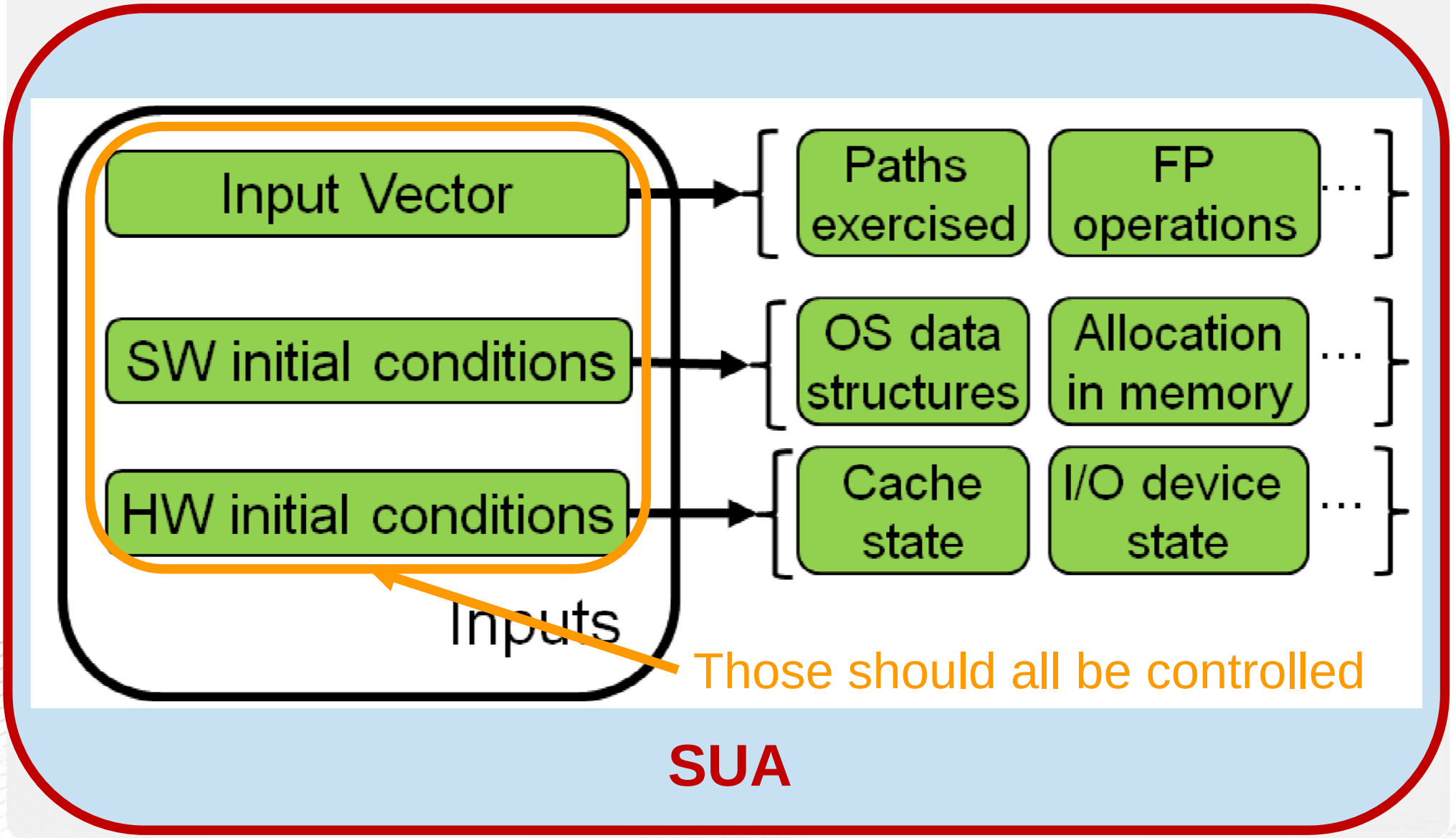
- *A possible solution is to look at the problem from the other end (the SUA)*
  - This requires first understanding and then – if possible – controlling the **sources of variability** in the outcome of observations
- *This is the premise of **Measurement-Based Probabilistic Timing Analysis** (MBPTA)*



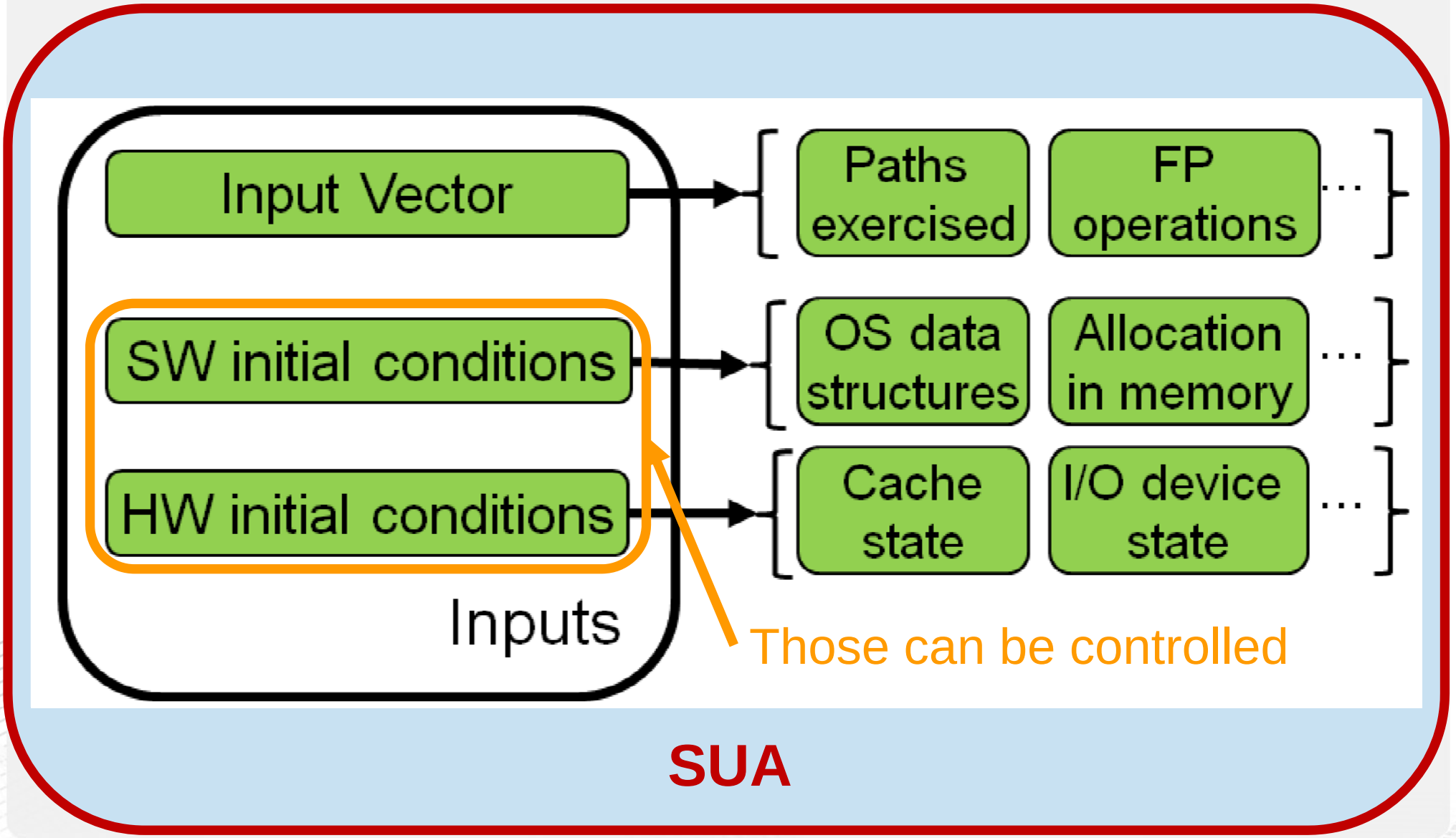
EVT in the WCET Context /4

- *In fact we are not interested in **all** elements of the universe of observations!*
- *We can «help» the analysis procedure using **block maxima** and concentrate on the sub-universe of maximal elements*
- *Since we cannot tell that sub-universe apart a priori we have to gear the SUA so that it does produce them*
- *In doing so we must ensure that the SUA is set to operational conditions*

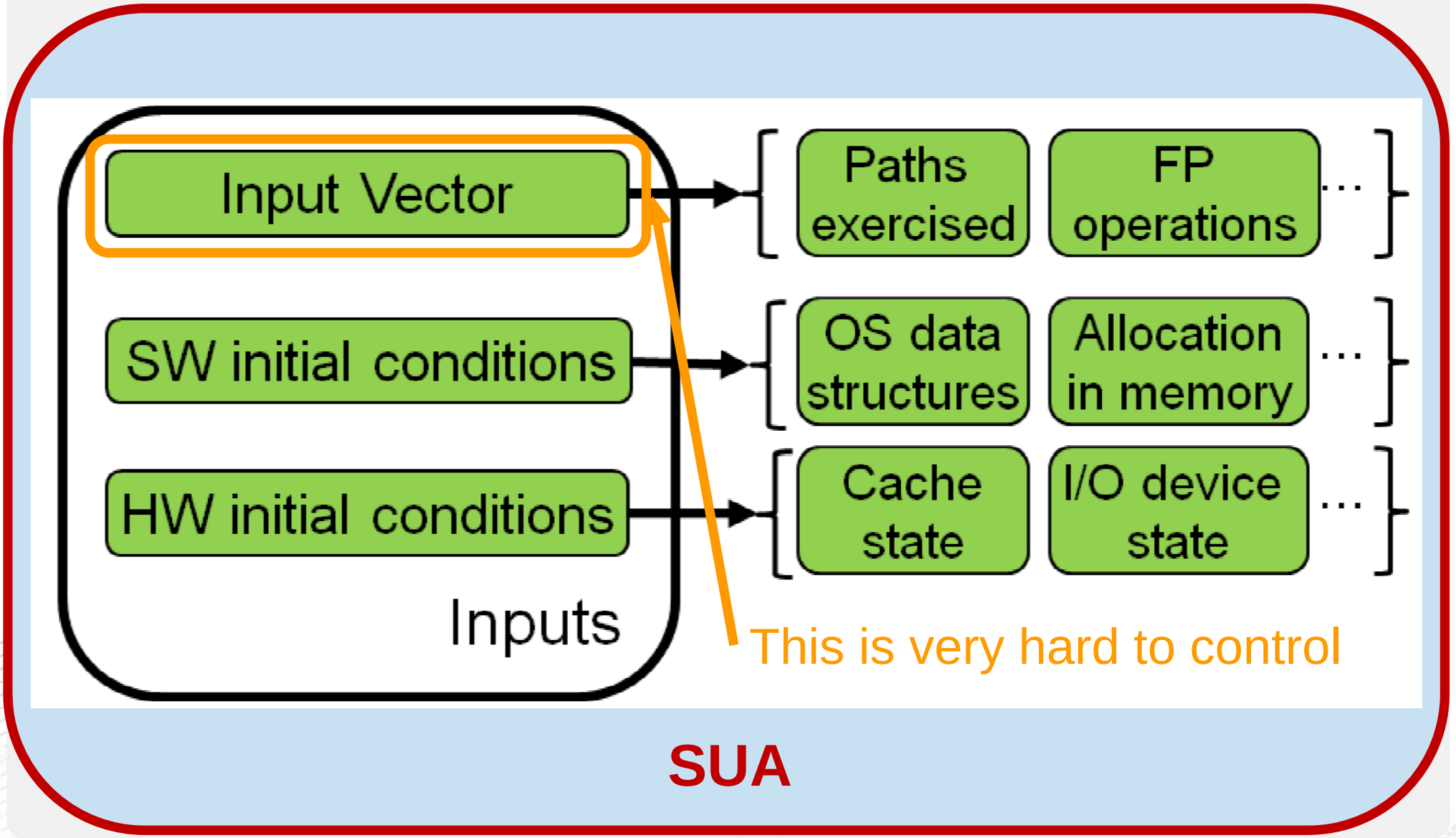
EVT in the WCET Context /5a



EVT in the WCET Context /5b



EVT in the WCET Context /5c



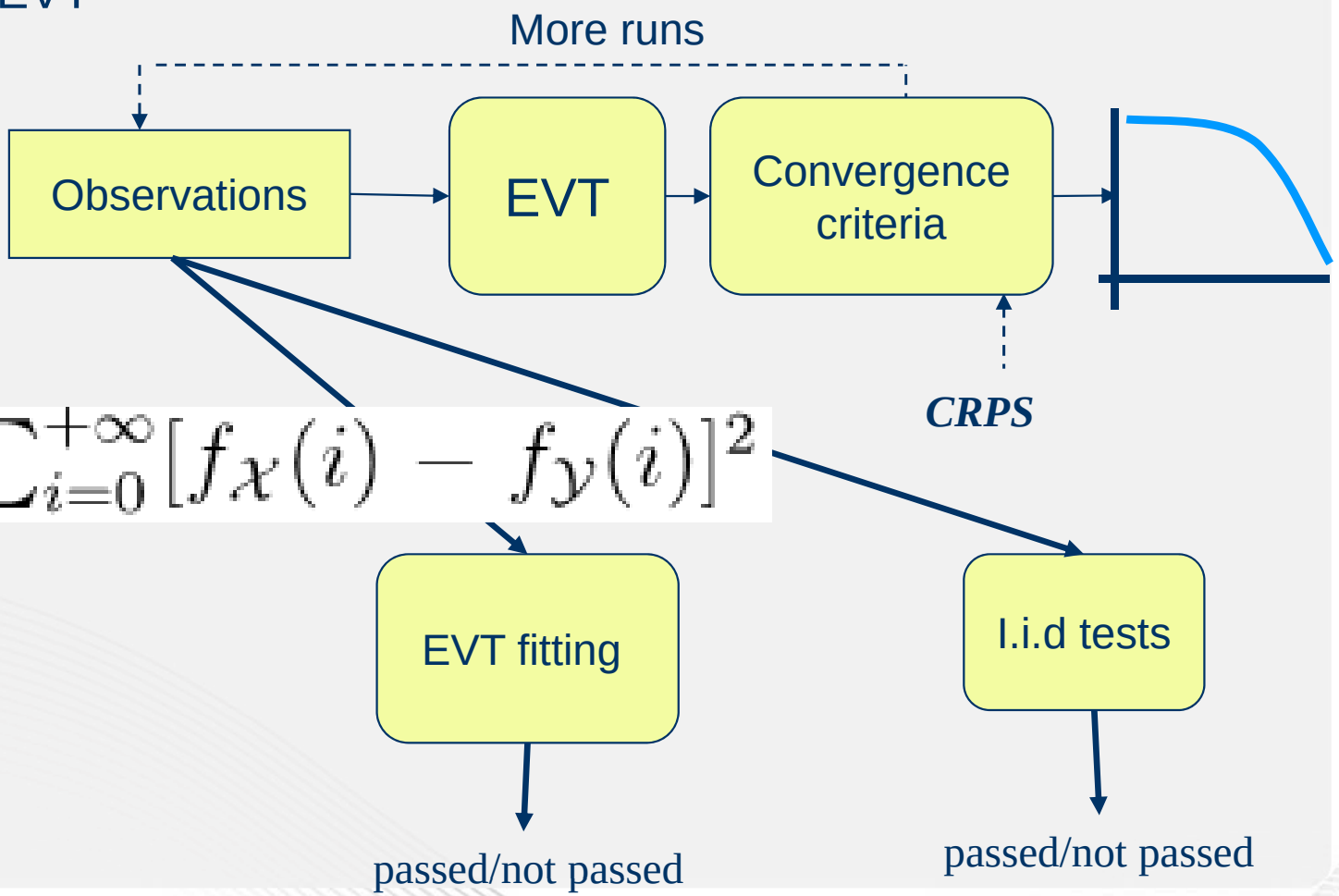
Time randomisation helps

- *Lots of sources of variability are hard for the user to effectively control from outside of the SUA*
- *A number of them can be written off by enforcing constant-time behaviour at the lower tiers of the execution stack underneath the application*
  - This can be done for the Operating System
  - This can be done by setting **low-jitter** processor resources to operate in worst-case mode
- *Or else injecting time randomization in **high-jitter** execution resources*
    - A change in the generation of latency not in the functional logic

EVT-MBPTA in a Nutshell

- ***Procedure***
  - Observations
  - Sampling
  - Fitting
  - Comparison
  - Tail extension

EVT



- ***Convergence***

$$CRPS = \sum_{i=0}^{+\infty} [fx(i) - fy(i)]^2$$

*ECRTS 2012

Take-Home Message

- *Extreme Value Theory is interesting*
- *It lends itself well to the WCET problem*
- *But its application needs (extreme ☺) care and attention*

- *PROARTIS has pioneered an EVT-MBPTA method that works well with «PTA-friendly» processors and time-composable Operating Systems*
- *You may want to read about it at http://www.proartis-project.eu/publications*

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